The size effects on the mechanical behaviour of fibres

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The size of a fibre affects its mechanical properties and thus is of theoretical and practical importance for studies of the rupturing process during loading of a fibrous structure. This paper investigates the overall effects of length on the mechanical behaviour of single fibres. Four types of fibres, ranging from brittle to highly extensible, were tested for their tensile properties at several different gauge lengths. Different from most previous studies where the focus has been on the gauge length effects on a single property such as fibre strength or breaking strain, this paper look comprehensively into the effects of length on all three of the most commonly studied mechanical properties, namely strength, breaking strain and initial modulus. Particular emphasis is placed on initial modulus and on the interactions between all three parameters. Influences of strain rate and fibre type on the size effects are also investigated. The effect of potential fibre slippage on experimental error is examined. An image analysis method is used to measure the real fibre elongation in comparison to the same fibre elongation obtained directly from an Instron tester. Finally, a statistical analysis is carried out using the experimental data to test the fitness of the Weibull theory to polymeric fibres. This was done as the Weibull model has been extensively utilized in examining fibre strength and breaking strain, although it is supposed to be valid only for the so-called classic fibres to which more extensible polymeric fibres do not belong.

1. Introduction

The connection between the tensile properties of fibres, fibre bundles and twisted fibre bundles (yarns) has been a topic of extensive research for its theoretical and practical significance. A related question is the gauge length or size effects on the properties of these materials. The increasing applications of composites reinforced by fibres or fibrous prepregs have further stimulated research interests in all these problems.

Despite numerous studies, several issues still remain unsolved. First, in the existing publications on the tensile properties of various fibrous structures, especially fibre-reinforced composites, the fibre's initial modulus has always been treated as a constant, independent both of its length and of the fact that the fibre may be present as part of a fibre bundle embedded in the matrix of a composite and will experience fragmentation during extension of the material [1–8]. Furthermore, although gauge length effects have been studied by many researchers, they have focused mostly on strength [9–15]. Very few [6] have examined breaking strain, and no one, known to the authors, has checked both properties at the same time, let alone together with the initial modulus.

The effect of diameter on the initial modulus of glass fibres has been studied by several scientists. Murgatroyd [17] reported that for high Na_2O-containing glass fibres, when the fibre diameter increased from 18 to 100 μm, the fibre Young's modulus rose from about 45 to about 70 GPa, a change of almost 35%. Similarly Bateson [18] observed that the fibre Young's modulus increased with fibre diameter. Additional work done by Norman and Oakley [19] showed that the Young's modulus of glass fibre measured by the ultrasonic technique was about 69 GPa for diameters in the range 30 to 200 μm and decreased by about 7.5% for 6 μm diameters. Pahler and Bruckner [20] found that for pristine E-glass fibres, the modulus increased with increasing fibre diameter by about 20%.

Moreover, with the increasing awareness of the considerable impact that variations in fibre properties have in determining their mechanical behaviour and structures made from them, theoretical approaches to specify the statistical distribution of the fibre properties have become desirable [11, 12]. Many have turned to the Weibull distribution. This model has been shown to be valid for brittle fibres, as these fibres are close to the so-called "classic fibres" on which the Weibull theory for fibre strength was derived [9]. The application of this model has been expanded to polymeric fibres whose behaviour is notoriously time dependent, and has a high probability that some long-range property variations may exist due to manufacturing irregularities, thus violating the assumption of the classic fibre. Therefore, the validity of the Weibull model for these fibres has to be tested.

In addition, as pointed out in [5–7] for example, the effect of the gauge length has practical significance. It is well known that, in a fibrous structure under extension, fragmentation occurs prior to the failure of the structure. As a result, the fibres will eventually break into much shorter lengths. Since the properties of the overall structure are derived from the fibres, this much shorter effective fibre length leads to a completely different system behaviour than that predicted from the starting fibre length [23–24]. The discrepancy between the two is caused largely by the gauge length, or size effects, of the fibres.

In the present work, tensile tests have been carried out on single fibres, fibre bundles and yarns (fibre bundles with twist) at different gauge lengths to examine the effects of gauge length, and for yarns, of twist, on their breaking strain, strength and, in particular, initial modulus. The validity of the assumption that the initial modulus of these materials is independent of their length is examined, since if the fibre modulus is related to its diameter or transverse size as mentioned before, it is reasonable to suspect a connection between the fibre modulus and its length or its longitudinal size. Also the applicability of the Weibull model to ductile polymeric fibres is statistically tested.

In the first report of this comprehensive study, we focus on the behaviour of fibres. The results for both fibre bundles and yarns will be presented in a separate paper.

2. Experimental procedures

2.1. Sample description

Four types of filaments, polypropylene (PP), polyester (PET) and polyamide 66 (PA) (all supplied by BASF) together with carbon (obtained from Hercules) were chosen. Specifications of the products are provided in Table I, which are either provided by the manufacturer or converted from the manufacturer's data.

2.2. Sample preparation and test

All the specimens were prepared and tested according to the standard method ASTM D 2101-93 for single fibres.

All specimens of the same fibre type were taken from the same spool to avoid extra variation. Attention was paid not to damage the fibres, especially the carbon type, during the whole process of specimen preparation. Using the tabbing method, specimen ends were held using tapes for easy handling and to prevent or alleviate possible fibre slippage during testing. All specimens were conditioned, prior to testing, at 65% RH and 21 °C to assure first the specimens reached the environmental equilibrium and also the tapes were adequately cured so that the specimen ends were held securely.

The tests were carried out on an Instron machine model 1122 with computerized data acquisition and analysis software called the Instron Series IX Automated Materials Testing System. Among all the settings in the software, we selected the Yarn/Fibre Test Method for the test, and the Automatic Limit option for calculation of the initial modulus. The calculation as described according to user's manuals [25] "begins the search for the steepest linear region at the start of the testing curve and ends it either at the yield point or at the end of the test, whichever occurs first". For accuracy, the charter on the Instron machine was also utilized to record the test results, which were later used to check the data from the computer.

Four different gauge lengths were chosen, 10, 20, 50 and 100 mm, for the test. For the gauge length of 50 mm, at least 30 tests were completed for each determination, and for other gauge lengths, no less than 20 tests were performed each time. When mounting specimens onto the tester, special care was taken to prevent fibre misalignment.

TABLE I Fibre descriptions

	Fibre type			
	PP	PET	PA	Carbon
Fibre density, ρ_f(g cm^{-3})	0.91	1.36	1.14	1.79
Fibre Denier (g/9000 m)	3.830	5.222	2.144	0.810

2.3. Image analysis technique for fibre elongation measurement

A critical issue in this study is the question whether slippage exists during testing and, if so, what is the actual effects on the testing results. To effectively investigate this problem, we employed a Sony colour video camera DXC-107A (CCD-IRIS) and a video recorder for image capturing and storing.

As there was no need to involve all four fibre types, we used PP fibres for the experiment. We selected a gauge length of 10 mm for this investigation since, if there was any fibre slippage, it would be most significant at this length.

For easy specimen manipulation and higher resolution, we decided after several trials to run two groups of tests. First, all fibre was tested at a constant strain rate at 100% min^{-1} using the Instron machine at gauge length 10 mm. Next we prepared fibres of gauge length 20 mm, and marked on the specimen a length span of 10 mm as our reference. These samples were also extended at the same strain rate to break on the Instron machine and the whole process was video-recorded using the image system. The video tape was then played on a Macintosh computer and the distances between the two marks initially and then right before the fibre broken were determined. This way the real fibre breaking extension and breaking strain were calculated. The results are provided in Table II.

3. Analysis of test results

3.1. Effect of fibre slippage on breaking strain

Although the strain values measured using the Instron tester were frequently higher than those by the image

TABLE II Fibre breaking strain (%) measured by the Instron tester and by image analysis for PP fibres

n	Instron tester	Imge analysis	x
1	28.14	25.00	3.14
2	24.64	22.73	1.91
3	23.58	27.78	− 4.20
4	28.91	20.83	8.08
5	31.81	21.74	10.07
6	24.16	19.23	4.93
7	24.96	27.27	− 2.31
8	27.27	25.00	2.27
9	22.39	20.83	1.56
10	22.83	30.43	− 7.06
11	18.93	20.00	− 1.07
12	34.70	23.81	10.89
13	29.01	21.74	7.27
14	23.21	22.73	0.48
15	29.63	21.74	7.89
16	35.11	21.74	13.37
17	22.58	36.36	− 13.78
18	25.15	23.08	2.07
19	27.27	20.00	7.27
20	23.54	17.39	6.15
21	26.62	19.23	7.39
Mean	26.40	23.27	$\bar{x} = 3.13$
S.D.	4.02	4.23	$s_x = 6.32$

analysis method in Table II, indicating that certain slippage indeed exists during measurement, it cannot be concluded that the two groups of data are significantly different. For that purpose, we utilize a statistical method testing the means for data of matched pairs [26]. First the differences x between each pair of the data points from the two populations were calculated, as well as the mean and the standard deviation, $\bar{x}$ and s_x, of x shown in Table II. Then according to the method, the t-statistics for the test can be calculated as

$$t = \frac{\bar{x}\, n^{1/2}}{s_x} = 2.270 \qquad (1)$$

where $n = 21$ is the number of data. At a value of 20 for the degree of freedom and a significance level of $\alpha = 0.01$, we can find in [26] a critical t value (t_α) of 2.528. Since $t < t_\alpha$, we can infer, in our measurement of fibre properties, that although there is some fibre slippage, the error it causes is statistically insignificant.

3.2. S–S curves of the four fibre types

The four fibre types selected had different stress–strain (S–S) curves (Fig. 1). It is clear that the PET fibre was undrawn, and the carbon fibre had an essentially perfect linear curve.

3.3. Constant rate of extension (CRE) versus constant rate of strain (CRS)

First, single fibre tests were done on PP with different gauge lengths at a constant strain rate at 100% mm^{-1} (Table III). It was observed that as the gauge length was increased breaking strain dropped markedly and strength slightly while initial modulus increased.

However our major concern was the consequences of the fragmentation process during extension of a fibrous structure. In this process, although the length of fibre decreases as fragmentation occurs, the extension rate of the whole structure remains unchanged. Thus for a study of the *in situ* behaviour of the fibres in the structure, testing at constant rate of extension not constant rate of strain seems more relevant. Therefore, further single fibre tests, on all our fibre types, were carried out at a constant rate of extension by setting the crosshead speed at 20 mm min^{-1}. Thus, for fibres with different gauge length, the strain rate changed, and we will compare this impact in the next section. The data are given in Table IV. A constant rate of extension of 20 mm min^{-1} is in fact equivalent to a constant rate of strain 100% min^{-1} when the sample length is 20 mm. Thus, to avoid unnecessary repetition of testing, we transferred the PP data for 20 mm gauge length from Table III to Table IV.

3.4. Gauge length effects on fibre breaking strains and strengths and the strain rate influence

The results for PP fibres at both CRS (Table III) and CRE (Table IV) yielded the same trend in terms of gauge length effects on fibre properties. Fibre breaking strain and strength increased as the gauge length decreased in both cases. The standard deviations of the fibre strength and breaking strain also increased, in general, for a reduced fibre length, albeit not as consistent as their mean values.

The comparisons between the data CRS and CRE are illustrated in Fig. 2. In Fig. 2a, it is seen that the breaking strain, ε_{fu2}, tested at a constant rate of extension of 20 mm min^{-1} is always lower than ε_{fu1} tested at a constant rate of strain of 100% min^{-1}. This seems to be in agreement to the experimental result given in [27] that, for synthetic fibres, a very low or very high strain rate will lead to a reduced breaking strain.

When strain rate increases, the tensile strength of a polymer material will increase as expected. Therefore, at long gauge length, a constant rate of extension corresponds to a lower strain rate, and hence a lower fibre strength as is the case in Fig. 2b. However, the effect of high strain rate appears to decrease at short gauge lengths, and may be hidden by experimental error.

As stressed above the purpose of this study was to investigate the effects of reduced fibre length due to the fragmentation process in a structure under extension. Thus, even though a structure may be under a tensile load at constant strain rate, a reducing fibre fragment length will alter the strain rate, making it closer to a case of constant rate of extension. Therefore, it appeared more rational here to carry out the tests at a constant rate of extension by fixing the crosshead speed at 20 mm min^{-1}.

3.5. Gauge length effects on fibre initial modulus

The gauge length influenced the initial modulus also for all four fibre types (Table IV and Fig. 2c). More

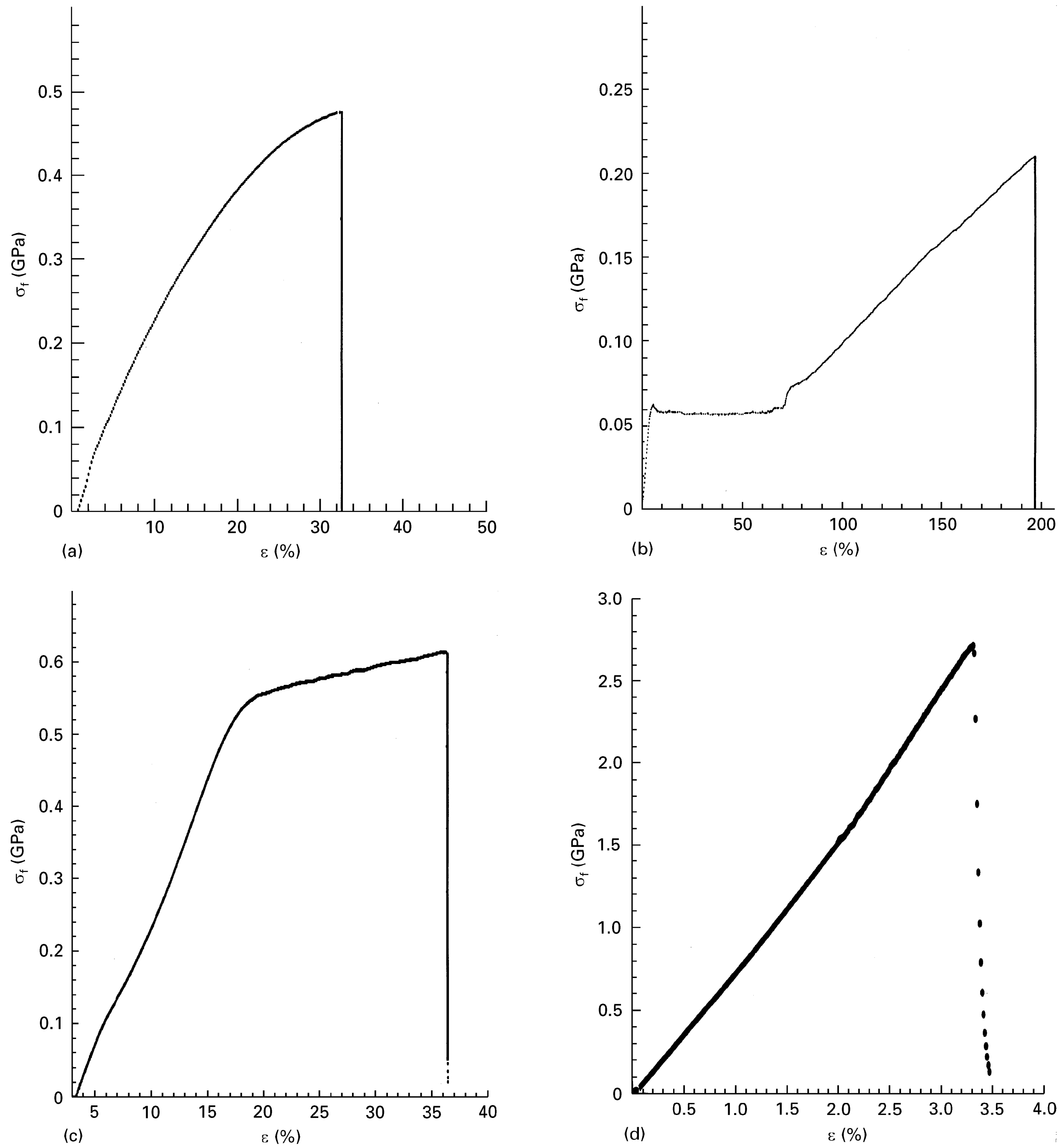


Figure 1 Typical stress–strain curves obtained at a crosshead speed of 20 mm min^{-1} and using a gauge length of 10 mm: (a) polypropylene fibre; (b) polyester fibre; (c) polyamide 66 fibre; and (d) carbon fibre.

TABLE III PP fibre properties and their SD values (in parenthesis) tested at constant strain rate (CRS) of 100% min^{-1}

	Gauge length (mm)			
	10	20	50	100
Breaking strain (%)	29.42 (3.66)	23.29 (3.46)	19.30 (2.74)	18.58 (2.74)
Strength (GPa)	0.382 (0.016)	0.375 (0.036)	0.373 (0.022)	0.371 (0.021)
Initial modulus (GPa)	2.100 (0.223)	3.585 (0.607)	4.688 (0.683)	4.772 (0.470)

specifically, in Fig. 3a where two stress–strain curves for PP fibre tested at two different gauge lengths are presented, one can actually see that the fibre initial moduli in these two cases are different. This finding is contrary to the widely accepted assumption that the fibre modulus remains constant as the fibre length changes. For all four fibre types, and in particular for polypropylene fibre in both CRS and CRE cases, the

TABLE IV Fibre properties and their SD values tested at constant rate of extension (CRE) of 20% mm min^{-1}

	Gauge length (mm)			
	10	20	50	100
PP				
breaking strain (%)	27.07 (4.55)	23.29 (3.46)	17.72 (2.72)	13.11 (2.08)
strength (GPa)	0.381 (0.054)	0.375 (0.036)	0.361 (0.033)	0.304 (0.035)
initial modulus (GPa)	2.658 (0.328)	3.585 (0.607)	4.837 (0.322)	5.104 (0.346)
PET				
breaking strain (%)	195.2 (19.21)	174.2 (5.82)	138.2 (9.73)	131.4 (5.50)
strength (GPa)	0.304 (0.016)	0.302 (0.014)	0.295 (0.020)	0.285 (0.018)
initial modulus (GPa)	2.276 (0.175)	2.576 (0.187)	3.077 (0.149)	3.382 (0.181)
PA				
breaking strain (%)	36.33 (3.75)		30.66 (12.75)	27.29 (6.77)
strength (GPa)	0.619 (0.026)		0.549 (0.054)	0.545 (0.033)
initial modulus (GPa)	4.258 (0.311)		4.458 (0.390)	5.130 (0.378)
Carbon				
initial modulus (GPa)	3.32 (0.64)	2.08 (0.34)	1.30 (0.44)	0.87 (0.31)
strength (GPa)	2.72 (0.43)	2.35 (0.24)	1.84 (0.75)	1.58 (0.55)
initial modulus (GPa)	86.35 (4.34)	125.01 (7.74)	147.27 (12.86)	185.44 (8.28)

fibre initial modulus decreased as the gauge length reduced. In other words, shorter fibres appear to be less stiff, a conclusion seemingly hard to comprehend.

There could be a number of potential causes responsible for this gauge length and fibre initial modulus connection. We can neglect the influence of fibre slippage during testing as discussed above. First, the specially selected and properly functioning specimen clamps and tapes, together with carefully monitored testing procedures have minimized the fibre slippage to being negligible. Also more importantly, since the extension loads applied to the fibres in the initial modulus region are very small, any slippage that might occur would be inconsequential.

Other possible causes for this modulus change include the strain rate effect. However as can be observed in Table III, it does not influence the trend of this initial modulus–length dependence on gauge length. Another is the effect of deformation of the clamps when stretching the fibres. However, because the clamps are much less deformable than the fibres, it will remain essentially constant for all the testings at different gauge lengths, and hence cannot be responsible for the marked changes of the modulus we have seen.

Next, the effect of load transfer from the clamps to fibre can also be neglected since the fibre was gripped so tightly that the load transfer length on the fibre is too tiny to cause any noticeable change. Furthermore we tested the possible cause of "machine lag" because it takes time for the Instron machine to reach the testing speed. We mounted the fibre specimen in a curled form, so that by the time the specimen became straight, and equal to the designated gauge length, the machine had reached its intended speed. Still, we found that the results, thus obtained, were consistent with the original data.

A more likely explanation is the difference between the changes caused by gauge length variations on the fibre strain and stress by which modulus is defined as

$$E_f = \frac{\Delta\sigma}{\Delta\varepsilon} \tag{2}$$

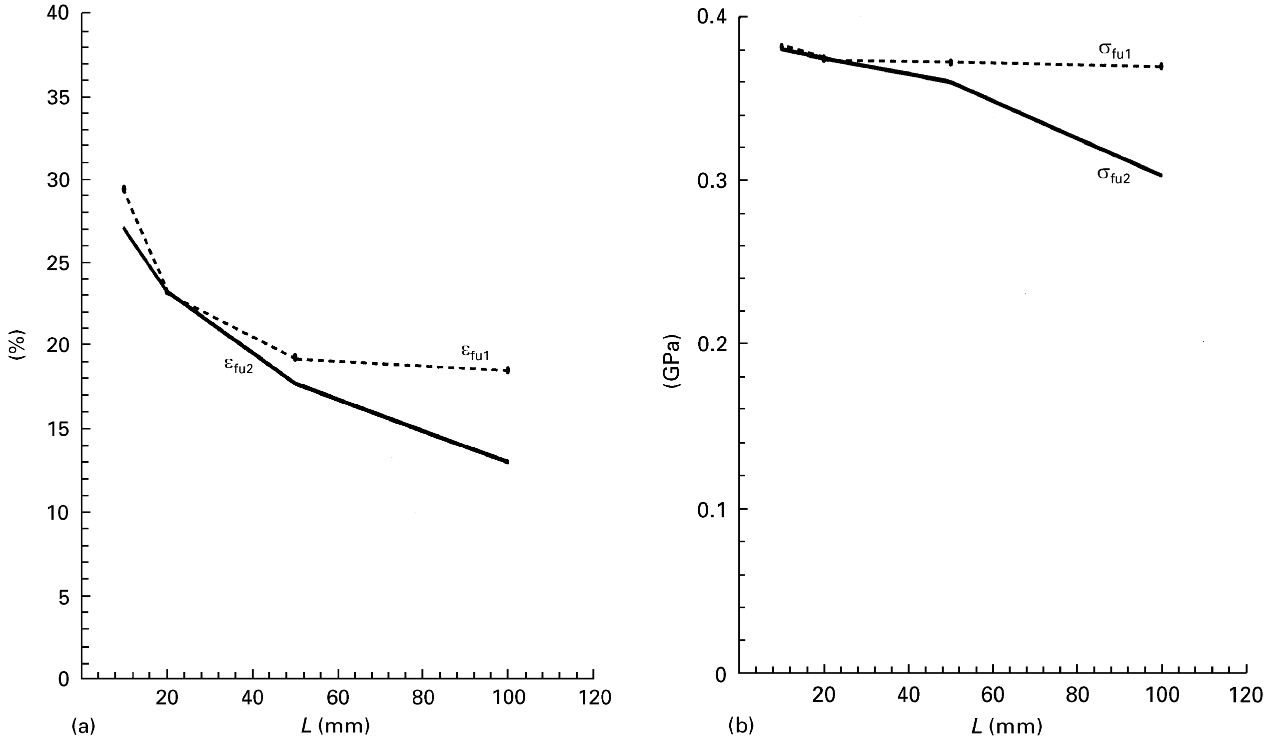
From the results in Tables III and IV, it is seen that although a reduction of gauge length will increase both fibre breaking strain and strength, the increase in the breaking strain is greater, which leads to decreasing modulus. Fig. 3b, a plot of three parameters normalized by their corresponding maximum values, illustrated this fact clearly over the range of gauge length tested for PP fibre.

From Fig. 3b we can conclude again that when gauge length decreases the fibre initial modulus declines. In other words, a smaller size leads to a smaller modulus, a conclusion in agreement with the findings on diameter effect of glass fibres [17–20]. Moreover, Fig. 3b also shows that when the fibre length changed from 10 to 100 mm, its modulus increased by over 40%, a magnitude consistent with that in [17] for the fibre diameter effect.

The above discussion also reveals an important point that, for a given material, the fibre modulus is not an independent statistical variable anymore once the strength and breaking strain of the material are given.

It should be noted that Morton and Hearle [28] have already claimed that gauge length should influence the fibre modulus. However their conclusion is not correct as seen from their analysis provided below. Suppose a fibre specimen consists of n sections, each of length x, and the extensions of the sections are represented by δx, varying from section to section at a constant stress level σ_f. Then, if the modulus is measured on this long specimen of length nx we have the fibre modulus

$$E_{f1} = \frac{\sigma_f}{(\sum \delta x)/nx} = \frac{\sigma_f x}{\langle \delta x \rangle} \tag{3}$$



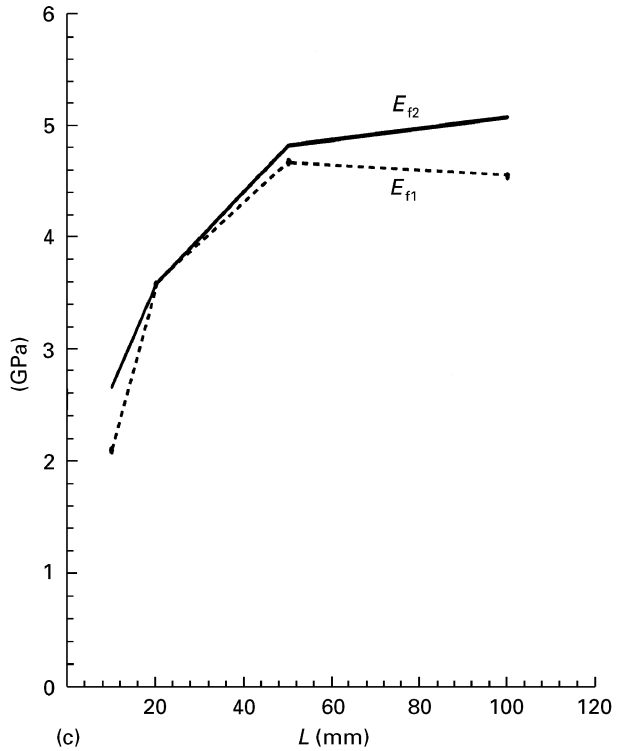


Figure 2 The strain rate effects on the polypropylene fibre properties: (a) strain rate effect on the fibre breaking strain (1–constant strain rate; 2–constant extension rate) (b) strain rate effect on the fibre strength (1–constant strain rate; 2–constant extension rate), and (c) strain rate effect on the fibre initial modulus (1–constant strain rate; 2–constant extension rate).

where $\langle \delta x \rangle$ is the mean value of δx. But, if the modulus is measured on the shorter lengths x, and then averaged, we should have

$$E_{f2} = \frac{1}{n}\sum\left(\frac{\sigma_f}{\delta x/x}\right) = \frac{\sigma_f x}{n}\sum\left(\frac{1}{\delta x}\right) = \sigma_f x\left\langle\frac{1}{\delta x}\right\rangle \tag{4}$$

or

$$E_{f1} = E_{f2}\frac{1/\langle \delta x\rangle}{\langle 1/\delta x\rangle} \tag{5}$$

Inevitably

$$\frac{1}{\langle \delta x\rangle} \leqslant \left\langle\frac{1}{\delta x}\right\rangle \tag{6}$$

Consequently, we have

$$E_{f1} \leqslant E_{f2} \tag{7}$$

In other words, according to this analysis, a longer fibre (or a fibre of larger size) would have a smaller initial modulus, a conclusion in conflict with our experimental results. In addition, the conclusion in Equation 7 does not rule out the case that $E_{f1} = E_{f2}$, that is, the fibre modulus is independent of its length.

Finally, it has to be pointed out that the implications of this size effect on the initial modulus of a material are very significant. First, it reveals that the increases in both strength and breaking strain of a material with smaller size are the results accumulated from every point of the material, or the increases are distributed over the whole mass of the material. Also, the size effects influence the strength and breaking strain of the material to a different degree, causing the modulus of the material to change. More importantly, because of this size effect on the material modulus, all the theoretical models predicting the composite behaviour based on the chain of the sub-bundle model caused by the fragmentation process [4, 5, 22] have to be modified since all of them have adopted the assumption that the modulus of the fibre is length independent.

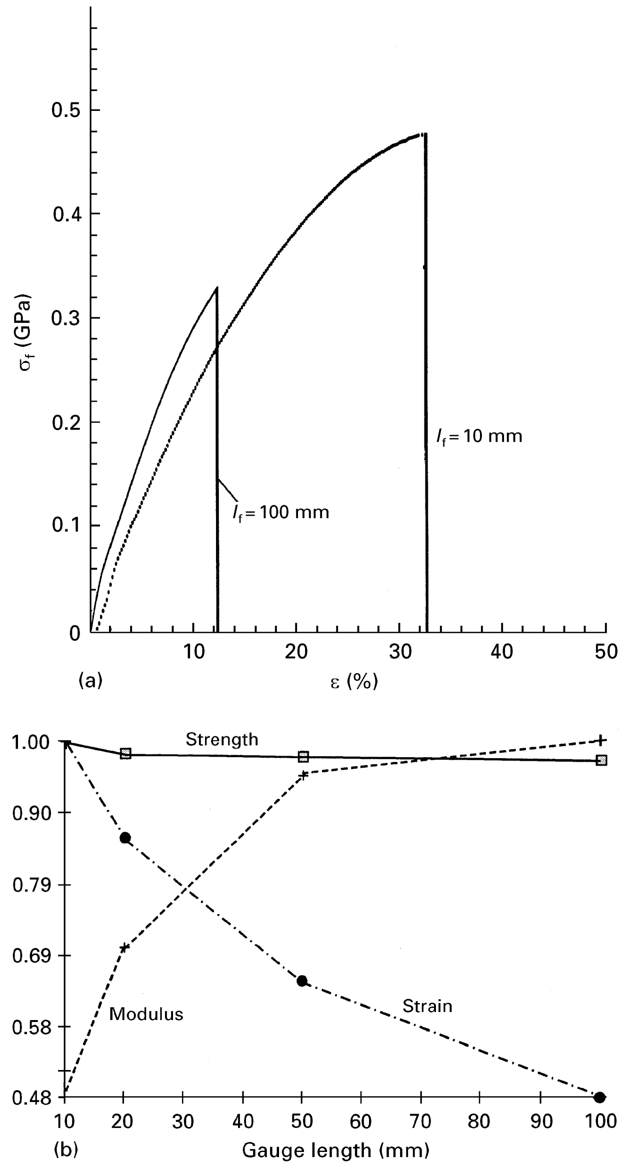


Figure 3 The gauge length effect on fibre initial modulus: (a) stress–strain curves at two different gauge lengths for PP fibre; and (b) changes of fibre strength, breaking strain and modulus with gauge length for PP fibre.

3.6. Comparison between fibre types

Since this size dependence of fibre initial modulus is observed for all four different fibre types, we suspect that this may be a universal law applicable to many materials just like the "Weakest Link" theory which has much less to do with the internal structure of a specific material. Yet more studies are needed before we can conclude that this phenomenon is related to the surface defects like the case of strength.

Nevertheless, we can still make a comparison of the size effects between all four fibre types used here. For that purpose, we have constructed Table V by normalizing all the data using their corresponding maximum values in Table IV for the different fibre types. We can then rank, for decreasing gauge length, all the four fibre types in terms of each property as follows:

breaking strain: carbon $<$ PP $<$ PET $<$ PA

strength: carbon $<$ PP $<$ PA $<$ PET

initial modulus: carbon $<$ PP $<$ PET $<$ PA

TABLE V Normalized fibre properties

	Gauge length (mm)			
	10	20	50	100
PP				
For breaking strain	1.00	0.86	0.65	0.48
For strength	1.00	0.98	0.95	0.80
For initial modulus	0.52	0.70	0.95	1.00
PET				
For breaking strain	1.00	0.89	0.71	0.67
For strength	1.00	0.99	0.97	0.94
For initial modulus	0.67	0.76	0.91	1.00
PA				
For breaking strain	1.00		0.84	0.75
For strength	1.00		0.89	0.88
For initial modulus	0.83		0.87	1.00
Carbon				
For breaking strain	1.00	0.63	0.39	0.26
For strength	1.00	0.86	0.68	0.58
For initial modulus	0.47	0.67	0.77	1.00

TABLE VI Test for fibre property Weibull distributions

Fibre type	$\alpha(GPa)$	β	d_n
Test for strength			
PP	0.375	13.158	0.112
PET	0.304	17.509	0.099
PA	0.572	12.700	0.076
Test for breaking strain			
PP	20.123	7.685	0.259
PET	142.475	17.336	0.062
PA	18.627	11.049	0.125

Each rank above reflects the size effect sensitivity of the different fibre types on a particular property. Based on their breaking strains, we can roughly characterize all four fibre types as brittle (carbon), moderate (PP), extensible (PET) and ductile (PA), respcetively. We can then conclude, based on the above ranks, that the sensitivity of a material's properties towards the size effect is closely related to its brittleness. The most brittle material will be the most sensitive to the size effect just as we expected.

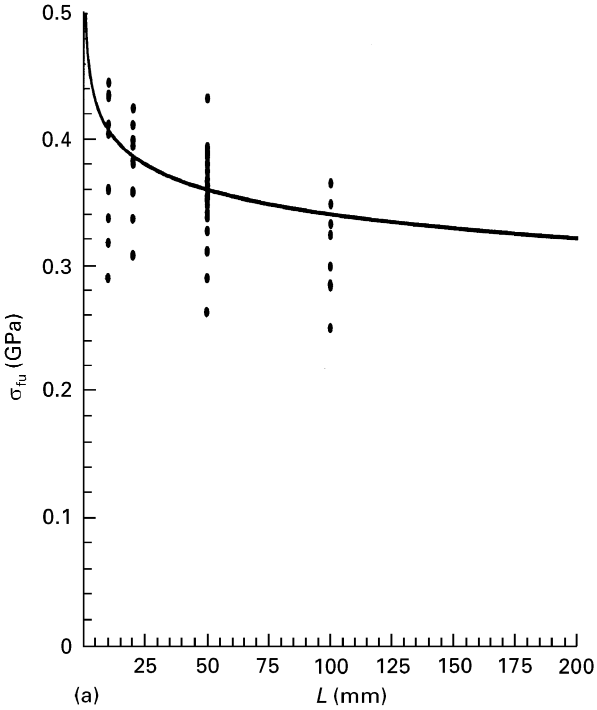
4. Test of the Weibull model using the fibre data

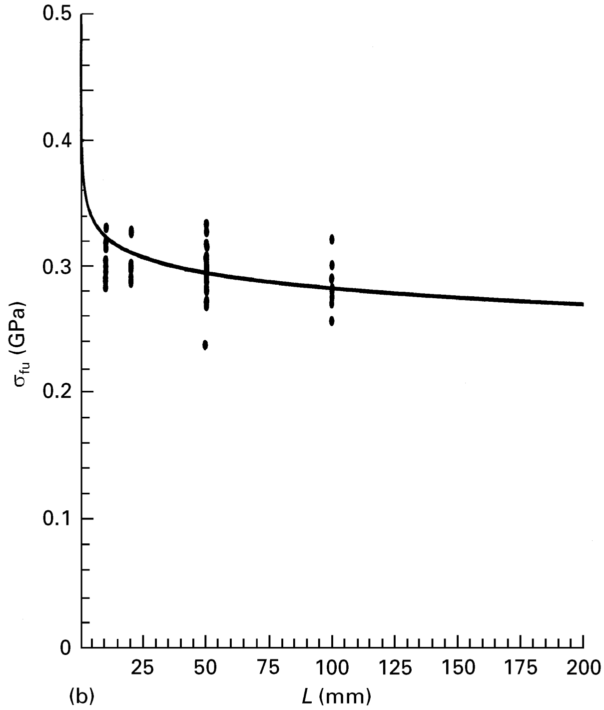
According to Daniels [29], fibre strength follows a two-parameter Weibull distribution, i.e. the probability of fibre strength being σ_f is given as

$$P(\sigma_f) \;=\; 1 - \exp\left[-\left(\frac{\sigma_f}{\alpha}\right)^{\beta}\right] \qquad (8)$$

where α is known as the scale parameter and β is the shape parameter of the fibre. Some researchers [16] assume that the fibre breaking strain is a Weibull-type variable as well.

In Daniels' analysis [29], the fibre were assumed to be classic fibres, i.e. brittle, and with time-independent fracture behaviour. However, all polymer fibres, including the ones used in this study, do not fall into this category. We will hence test this model using the data of the three polymeric fibres in Table IV.





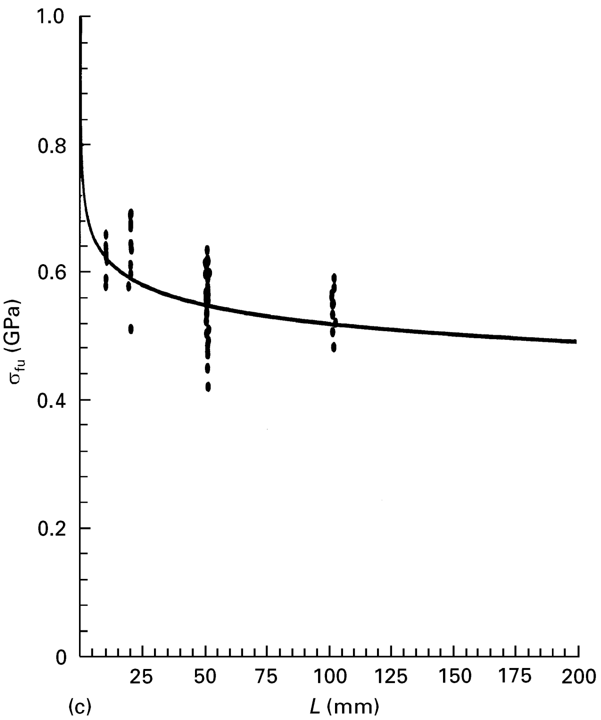


Figure 4 Weibull model and the experimental data: (a) polypropylene fibre ($\alpha = 0.375$; $\beta = 13.158$); (b) polyester fibre ($\alpha = 0.304$; $\beta = 17.509$); and (c) polyamide 66 fibre ($\alpha = 0.572$; $\beta = 12.700$).

First, we estimated the Weibull parameters of both strength and breaking strain of the fibres, using data tested at gauge length = 50 mm, and sample size $n = 30$, by means of the maximum likelihood estimation (MLE) technique, and the results for all three fibres are provided in Table VI.

We then plotted the Weibull curves of fibre strength versus fibre length using the estimated parameters for each fibre type, along with the experimental data at different fibre lengths in Fig. 4. Both the experimental and theoretical results indicate a trend of decreasing strength for longer fibre length. Yet, Fig. 4 is only a qualitative illustration. The fitness of the Weibull model to the experimental data has to be tested statistically.

To verify whether both strength and breaking strain of fibres obeyed the Weibull model, the Kolmogorov goodness-of-fit test [30] was used, following the procedures outlined below:

1. Rank all the experimental data x_i in an ascending sequence;
2. Calculate the sample statistical distribution using $(i - 0.5)/n$, where n is the total sample number;
3. Calculate the theoretical distribution according to Equation 8, using the estimated Weibull parameters;
4. Compare the corresponding pairs from 2 and 3, and find the maximum difference between them denoted as d_n;
5. Calculate the critical values of d_n: for significant level $a = 0.05$, $d_{nc} = 1.36/n^{1/2}$;
6. *If* $d_n < d_{nc}$, the statistical model is considered a good representation of the data distribution.

The d_n for both strength and breaking strength of the three types of fibres are thus calculated and listed in Table VI. The critical value $d_{nc} = 1.36/30^{1/2} = 0.2483$. We can hence conclude, by comparing d_{nc} with the d_n values in Table V, that the Weibull model is an adequate representation of the data distributions of both strength and breaking strain of the three polymeric fibre types, even though these fibres have considerably different stress–strain curves and do not meet the requirements for "classical fibres".

5. Conclusion

For the fibre types investigated in this study, the gauge length at which the fibre was tested affected its strength, breaking strain and, more importantly, initial modulus. It is common knowledge that a fibre becomes stronger and, less commonly known, more extensible as its length is reduced. However, contrary to the widely accepted assumption that fibre initial modulus is independent of its length, the results of this study have shown that the fibre initial modulus decreases as fibre length is reduced. In other words, a fibre of shorter length will become stronger, more extensible but less stiff in extension. This conclusion is valid regardless of the strain rate.

The implication of this size effect on fibre modulus is profound. First, it reveals that the increases in both strength and breaking strain of a material with smaller size are the results accumulated over every tiny segment of the whole fibre. Furthermore, the reason that the modulus of a fibre changes is due to the fact that size influences the strength and breaking strain of a fibre by different amounts. More importantly, because of this size effect on fibre modulus, the models predicting the composite behaviour based on the chain of sub-bundle model have to be modified because all of them have assumed that the modulus of the fibre is length independent. In addition, a more brittle material will be more sensitive to the size effect.

Also, it has been shown that the Weibull model is a good theory to specify the statistical distributions of both strength and breaking strain of polymeric fibres, irrespective of the fact that these fibres do not satisfy the definition of a "classic fibre" originally adopted as assumptions in deriving the Weibull model.

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